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Turbulent acoustic streaming excited by resonant gas oscillation with periodic shock wave in a closed tube

Takeru Yano

Department of Mechanical Science, Hokkaido University, Sapporo, 060-8628, Japan

e-mail: yano@lebes-me.eng.hokudai.ac.jp

Abstract: The fundamental resonant gas oscillation with periodic shock wave in a closed tube is studied by executing large-scale computations of the 2-D Navier–Stokes equations for compressible flow with a high-resolution upwind finite-difference TVD scheme. In a quasi-steady state of gas oscillation, acoustic streaming (mean mass flow) with large Rs is excited in the tube, where Rs is the streaming Reynolds number based on a characteristic streaming velocity, the tube length, and the kinematic viscosity. In the case of Rs $= 560$, relatively strong vortices are localized near the tube wall, and the resulting streaming pattern is almost stationary but quite different from that of the classical Rayleigh streaming. Streaming of Rs $= 6200$ involves unsteady vortices in a region around the center of the tube. Turbulent streaming appears in the result of Rs $= 56000$, where a lot of vortices of various scales are irregularly generated throughout the tube.

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Introduction

We shall consider streaming motions excited by the fundamental resonant gas oscillation in a two-dimensional closed tube filled with an ideal gas. The tube, whose length is L and width is W, is closed at one end by a solid plug and the other by a piston (sound source) oscillating harmonically with an amplitude a and angular frequency ω (see Fig. 1). When the source frequency is in a narrow band around a resonant frequency, the resulting gas oscillation may not be a sinusoidal standing wave with fixed loops and nodes but a nonlinear oscillation of large amplitude including periodic shock waves traveling in the tube repeatedly reflected at the sound source and closed end.[1]

Such large-amplitude oscillations can induce streaming motions of large streaming Reynolds number Rs $= U_S L_S/\nu$, where U_S is a characteristic magnitude of streaming, $L_S(= L)$ is a linear dimension of the system, and ν is the kinematic viscosity. For Rs $\gg 1$, streaming in the tube can become a turbulent flow, as the jet-like streaming.[2,3] In the present letter, we shall numerically demonstrate when and how the streaming motion in the tube becomes turbulent. Since very large-scale computations are required, we restrict ourselves to the case that the angular frequency at the sound source, ω, is equal to the fundamental resonance (angular) frequency $c_0\pi/L$, where c_0 is the speed of sound in an initial undisturbed gas.

In practical applications of high-intensity resonant oscillations in a closed tube, an available model of streaming has so far been limited to that induced by the linear sinusoidal standing wave with fixed loops and nodes; however, it should be a steady creeping motion of Rs < 1.[3–6] Understanding of turbulent streaming may be indispensable to the development of such applications.

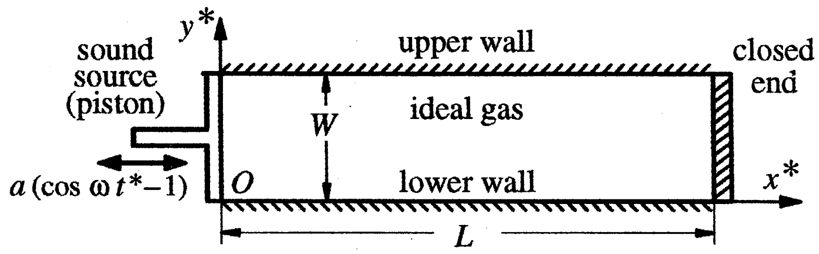

Fig. 1. Schematic of the model.

Problem

We shall consider the fundamental resonance in the closed tube in a case of wide tube and low frequency. That is, the tube length L satisfies the condition

$$\frac{L\omega}{c_0} = \pi, \tag{1}$$

and the width W is sufficiently large compared with a typical dimension of the Stokes boundary layer on the wall,

$$\frac{W\omega}{c_0} = A\pi \gg \epsilon = \frac{\sqrt{\nu\omega}}{c_0}, \tag{2}$$

where $A = W/L$ is an aspect ratio of tube, ϵ is a normalized typical linear dimension of the Stokes boundary layer on the wall.

The sound source is a piston located at $x = 0$ for $t < 0$, which begins oscillating harmonically with amplitude a and angular frequency ω at $t = 0$, where $x = x^*\omega/c_0$ and $t = \omega t^*$. The acoustic Mach number at the source, M, is supposed to be sufficiently small compared with unity,

$$M = \frac{a\omega}{c_0} \ll 1. \tag{3}$$

In the present study, we assume that M is comparable with ϵ, i.e.,

$$\alpha = \frac{\epsilon}{M} = O(1), \tag{4}$$

where α is a nondimensional constant. The acoustic Reynolds number at the sound source may then be given as

$$\mathrm{Re} = \frac{2\pi}{\epsilon\alpha}, \tag{5}$$

which is sufficiently large compared with unity. In addition to condition Re $\gg$ 1, if the dispersion and attenuation effects[7] due to the Stokes boundary layer are sufficiently small, a shock with discontinuous wave front will be formed. The dispersion effect can be estimated by a nondimensional parameter $\epsilon/(A\sqrt{M})$, because, as shown below, a nondimensional wave amplitude at an almost steady state (quasi-steady state) is of $O(\sqrt{M})$.

Under the conditions (1)–(4), we shall numerically solve the initial- and boundary-value problem of the two-dimensional Navier–Stokes equations for compressible flow. We assume that the temperature on the solid surface is constant. The gas in the tube is considered to be air (the ratio of specific heats is 1.4 and the Prandtl number is 0.7). Sutherland's formula is adopted for the temperature dependence of shear viscosity, and the bulk viscosity is neglected for simplicity. The flow field is supposed as symmetric around $y = A\pi/2$, where $y = y^*\omega/c_0$.

Numerical method

We need to use a numerical method capable of resolving discontinuous shock waves, and therefore an upwind finite-difference TVD scheme by Chakravarthy[8] is employed, since the capability of the method has already been confirmed in the analysis of the near field of oscillating circular piston.[9] The 2-D Navier–Stokes equations are directly solved without introducing any further assumptions. The turbulent streaming motion is not artificially excited but self-generated in the numerical solution for the case of sufficiently large Rs.

The lower half of the tube, $M(\cos t - 1) \leq x \leq \pi$ and $0 \leq y \leq A\pi/2$, is subdivided into a 300×60 nonuniform mesh, where the minimum grid size is less than $\epsilon/4$. Mesh points are clustered near the lower wall, sound source, and closed end, and hence we can resolve the Stokes boundary layer and a secondary boundary layer[2,3] of thickness of $O(1/\sqrt{\mathrm{Rs}})$. The time step is $2\pi/120000$ and the CFL number is about 0.5. The cpu time for 250 periods of oscillation of piston exceeds 200 hours on the supercomputer at Hokkaido University.

The important parameters which characterize the present problem are the source Mach number M, the normalized thickness of Stokes layer, ϵ, and the aspect ratio A: $A = 0.1$ and ϵ is chosen as 4.5×10^{-4}. The latter corresponds to source frequency $\omega/2\pi = 250$Hz in the air of the standard state. We have computed three cases of $M = 0.000036$, 0.0004, and 0.0036. The parameters and results are summarized in Table 1. Short animations of main results can be seen at URL: http://www.hucc.hokudai.ac.jp/~b11422.[10]

Table 1. Parameters and main results.

Mach number at Source	SPL* (Source)	SPL** (Max)	$\mathrm{Rs} = \dfrac{\pi}{\alpha\epsilon}$	$\dfrac{\epsilon}{A\sqrt{M}}$	Shock	Streaming
0.000036	104.9dB	147.5dB	560	0.75	finite thickness	stationary flow pattern
0.0004	125.8dB	161.9dB	6200	0.23	discontinuity	unsteady
0.0036	144.9dB	172.4dB	56000	0.08	discontinuity	turbulent

* SPL for the plane progressive wave radiated by the corresponding sound source.
** SPL based on the rms value of pressure perturbation at closed end in the quasi-steady state.

Resonant gas oscillation with periodic shock waves

First of all, we shall present the evolution of on-axis velocity amplitude from the initial state of uniform and at rest (Fig. 2). The amplitude initially grows in proportion to Mt. At a large t of $O(1/\sqrt{M})$, an almost steady state (quasi-steady state) is established, where the maximum amplitude of oscillation during one period is almost constant of $O(\sqrt{M})$. The quasi-steady state is supported mainly because of the balance of energy input at the source and energy dissipation at the shock front.

Figure 3 shows the wave profiles in the quasi-steady state. A wire-framed yellow disk in the figure is merely a virtual image of sound source. Note that our computations are neither three-dimensional nor axisymmetric. Since $\epsilon/(A\sqrt{M})$ is not so small for $M = 0.000036$ (see Table 1), the dispersion effect due to boundary layer prevents shock front from steepening [Figs. 3(a) and 3(b)]. For the cases of $M = 0.0004$ and 0.0036, the shock front develops into a discontinuity. From Figs. 3(a), 3(c), and 3(e), one can readily see that the profile of axial fluid velocity has a small peak in the boundary layer (Richardson's annular effect).

Roughly speaking, the fluid motion outside the boundary layer can be regarded as the superposition of resonant oscillation and streaming motion. Accordingly, in the case that streaming velocity is relatively large and irregular (see Figs. 4 and 5), the axial velocity outside the boundary layer is slightly uneven as shown in Figs. 3(c) and 3(e). Since entropy (and also vorticity) is convected by streaming, the profiles of density and temperature also possess the same unevenness. Pressure profile, on the other hand, is hardly affected by the boundary layer and streaming, and hence it is almost independent of the distance from the lower wall.

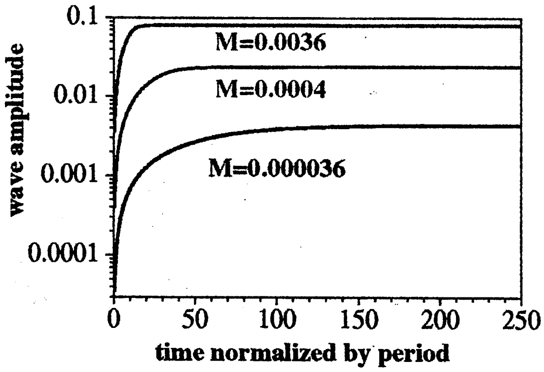

Fig. 2. Evolution of wave amplitude. The wave amplitude on the symmetric axis is evaluated as maximum of u minus minimum of u at $t = n\pi/2$ ($n = 0, 1, 2, ...$), where $u = u^*/c_0$ is the axial component of nondimensionalized fluid velocity.

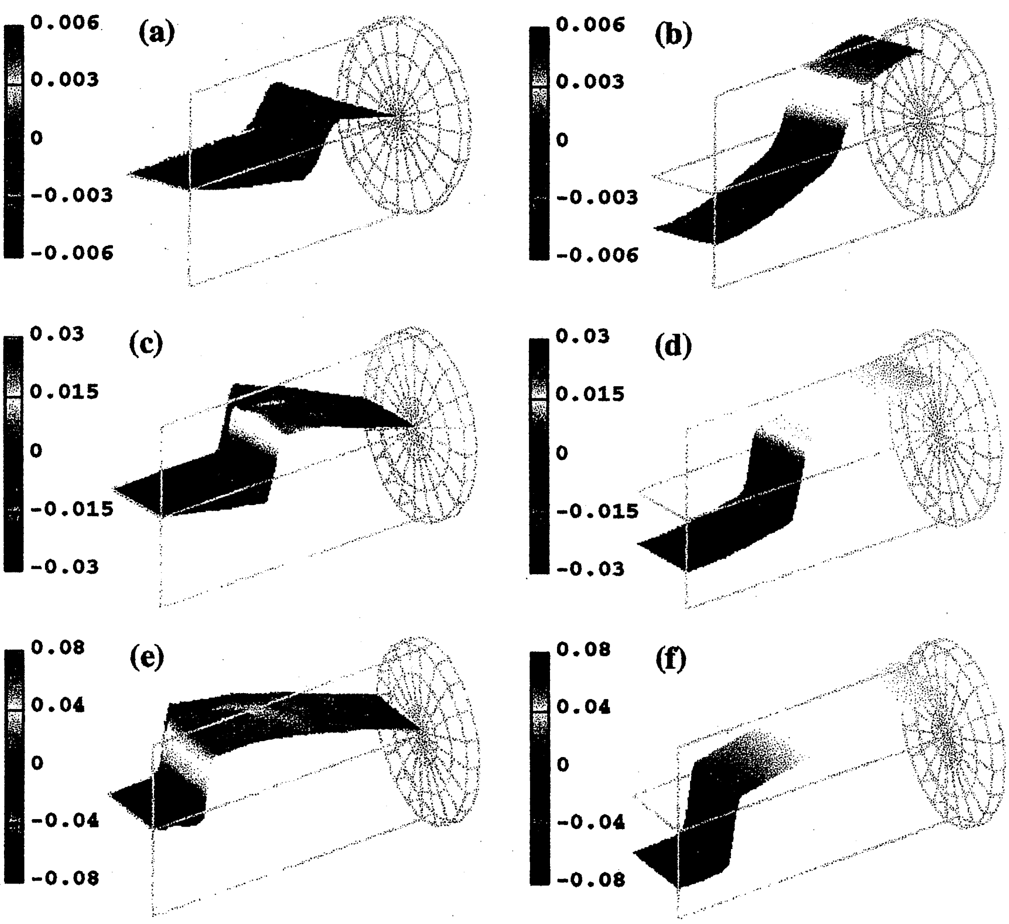

Fig. 3. Snapshots of wave profiles of the normalized axial velocity $u = u^*/c_0$ and sound pressure $p = (p^* - p_0)/\rho_0 c_0^2$. (a) u and (b) p for $M = 0.000036$ at $t = 879.5\pi$, (c) u and (d) p for $M = 0.0004$ at $t = 589.5\pi$, and (e) u and (f) p for $M = 0.0036$ at $t = 461.5\pi$. Each color bar indicates the value of u or p.

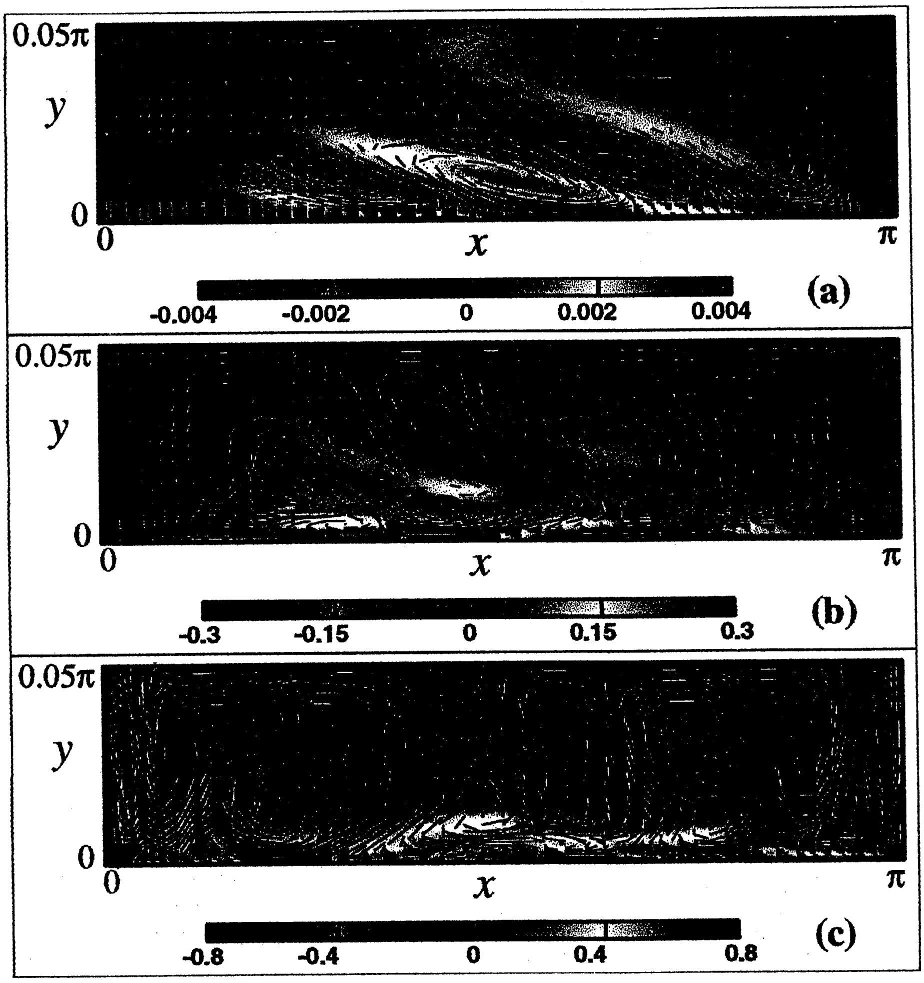

Fig. 4. Streaming patterns. The color bar denotes the strength of vorticity of streaming. (a): $M = 0.000036$ at $t = 918\pi$ (Rs = 560), (b): $M = 0.0004$ at $t = 628\pi$ (Rs = 6200), (c): $M = 0.0036$ at $t = 500\pi$ (Rs = 56000).

Excitation of turbulent streaming

The normalized velocity components of acoustic streaming (mean mass flow) are defined as

$$\boldsymbol{u}_S = \begin{pmatrix} u_S \\ v_S \end{pmatrix} = \begin{pmatrix} \overline{\rho u} \\ \overline{\rho v} \end{pmatrix} = \frac{1}{2\pi}\int_{t-2\pi}^{t} \begin{pmatrix} \rho u \\ \rho v \end{pmatrix} dt, \tag{6}$$

where $\rho = \rho^*/\rho_0$ is a normalized density, $u = u^*/c_0$ and $v = v^*/c_0$ are x and y components of the normalized fluid velocity, and the bar denotes the time average. A typical streaming velocity in each case is of $O(M)$, i.e., square of the maximum fluid velocity of $O(\sqrt{M})$. The nominal streaming Reynolds number Rs can therefore be estimated as $\pi/\alpha\epsilon$ (see Table 1). We have numerically confirmed that, as in the linear standing wave problem, u_S and v_S are nearly equal to $\overline{u}$ and $\overline{v}$, because oscillation of ρ is out of phase with that of fluid velocity and hence the so-called velocity transform[6] is small compared with the magnitude of $\boldsymbol{u}_S$. We have also confirmed that $\operatorname{div} \boldsymbol{u}_S = \operatorname{div} \overline{\rho \boldsymbol{u}} = O(M)$ in all cases.

The color contours in Fig. 4 indicate the distribution of $\operatorname{curl} \boldsymbol{u}_S$. In the case of $M = 0.000036$ (Rs = 560), the streaming pattern shown in Fig. 4(a) is almost invariant from $t = 440\pi$ to at least $t = 918\pi$. Since Rs is not small, vorticity in the Stokes layer is hardly diffused and, in addition, the streaming velocity is not large enough to propagate the vorticity in the vicinity of wall to all over the tube. As a result, some strong vortices are localized near the wall and the flow pattern in Fig. 4(a) is quite different from that of the classical slow streaming of Rs < 1 excited by the linear standing wave.[3–6] Figure 4(b) shows the streaming pattern for $M = 0.0004$ (Rs $= 6200$), in which several unsteady vortices are produced and they are confined in a region around the center of the tube, at least up to $t = 628\pi$. Turbulent streaming appears in the case of $M = 0.0036$ (Rs $= 56000$), where irregular and unsteady vortices of various scales are produced throughout the tube [see Fig. 4(c) and Fig. 5(c)].

Figure 5 shows the temporal evolution of u_S/M. Comparing with Fig. 2, one can see that u_S at $x \cong \pi/2$ abruptly grows when the oscillation attains the quasi-steady state. We here remark that u_S is very small at $x \cong \pi/2$ in the classical streaming of Rs < 1.[3–6]

In the case of Rs $= 560$, although the streaming pattern shown in Fig. 4(a) is almost stationary, the local streaming velocity shown in Fig. 5(a) gradually varies for $t > 500\pi$. The axial streaming velocity for Rs $= 6200$ in Fig. 5(b) is unsteady after the oscillation has reached the quasi-steady state. However, we cannot examine whether the fluctuation of streaming is irregular or not because the numerical result for Rs $= 6200$ is limited to $t \leq 628\pi$; the required computation to answer the question is too large to be executed. In the case of Rs $= 56000$ [Fig. 5(c)], u_S fluctuates irregularly throughout the tube.

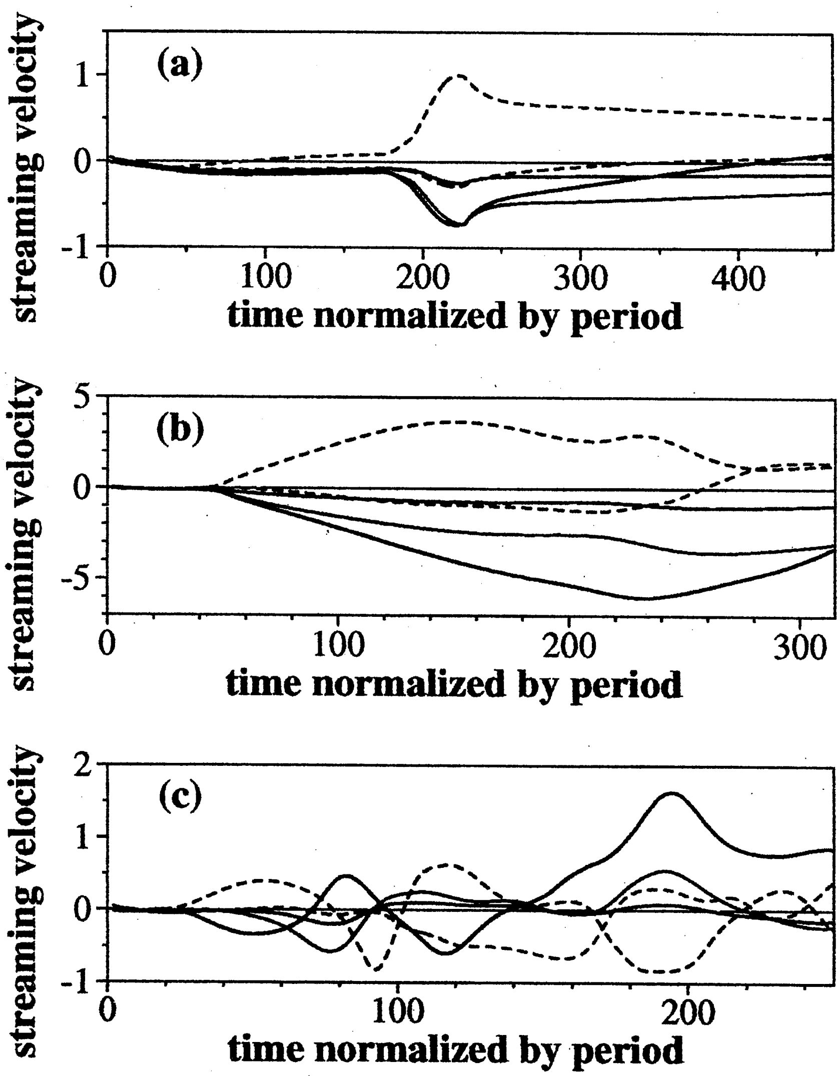

Fig. 5. Evolution of u_S/M at $x = 1.4084$ and $y = 0.0012$ (red solid curve), $y = 0.0043$ (green solid curve), $y = 0.0130$ (blue solid curve), $y = 0.0348$ (red dashed curve), and $y = 0.0821$ (green dashed curve). (a): $M = 0.000036$ (Rs $= 560$), (b): $M = 0.0004$ (Rs $= 6200$), (c): $M = 0.0036$ (Rs $= 56000$).

Conclusions

We have numerically demonstrated the excitation of turbulent acoustic streaming by the resonant gas oscillation in the closed tube. If the acoustic Reynolds number at the sound source Re is sufficiently large compared with unity, the resonant gas oscillation attains the quasi-steady state at a large t of $O(1/\sqrt{M})$, where the normalized wave amplitude is of $O(\sqrt{M})$. The magnitude of resulting acoustic streaming, which is induced by the second-order nonlinear effect of gas oscillation, is of $O(M)$, namely the streaming velocity is the same order of magnitude as the piston velocity at the sound source. Accordingly, Rs is as large as Re. Very large Rs flows can thus be realized. This leads to the occurrence of turbulent acoustic streaming.

Finally, we shall remark that, in the present computations, the Reynolds number based on the thickness of the Stokes layer is of $O(1)$ and streaming in the Stokes layer is disturbed but as a whole remains laminar. If Reynolds number based on the Stokes layer thickness exceeds its transition Reynolds number, the oscillation in the Stokes layer itself will become turbulent and the turbulence will occur in the form of periodic bursts followed by relaminarization in the same cycle of oscillation.[11]

Acknowledgments

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